Air column resonance frequencies and their dependencies in cylindrical tubes

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Abstract
The paper deals with the relationships of acoustical and mechanical lengths of cylindrical tubes and their resonance properties by using the BIAS measurement method. The tubes with different lengths and diameters were measured in constant and changed temperature conditions. The results are discussed and compared with formulas well known to the organ builders.

Introduction

The acoustical length question of the oscillating air column in wind musical instruments, especially in organ pipes was discussed and answered in the past by physicians (Bernoulli, Savart), acousticians (Helmholtz, Rayleigh) and organ-makers (Cavaillé-Coll, Frobenius), too. There are the general well-known Meyer's works. Rayleigh determined the open-end correction on cylindrical columns as a difference between the acoustical and mechanical lengths exactly as 0.41 of the tube diameter. In addition to correction for open-end with the most frequent value 0.3 D the correction for mouth influences open organ pipes, too. The sum of both corrections represents the real air column extension according to organ-maker „Faustregeln" („Fist rules") (1.5-1.7 D). But during tuning and voicing of organ pipes or with a changed position the influence of the tone already in the larger distance in comparison to correction 0.3D is very often observed. This problem inspired the idea of verifying current correction size by the input impedance measurement using BIAS (Brass Instruments Analysis System from IWK Wien) in the simplest case of one-end closed cylindrical tubes.

Experiments and results

Seventeen duralloy tubes with internal diameter 13.9 – 55.8 mm at the lengths 985, 656 and 327 mm were prepared for measurements. In comparison with organ pipes the diameter 55.8 mm with the length 327 mm corresponds to open Flute (tone c5) or closed Gedeckt (tone c4), with the length 656 mm corresponds to open Diapason (tone c4) or closed Kvintadena (tone c3), with the length 985 mm open Salicional (tone f3) or extremely narrow closed unspecified pipe (tone f2). The tubes were precisely cut at one end for perfect closed contact with the BIAS transducer. Following input conditions were defined: closing of tube, temperature and humidity inside and outside of tube, air flow and noise level.

The first control measurement of frequency dependence on air temperature was performed for 10 resonance modes with the tube length 985 mm, for 8 modes with the length 656 mm and for 5 modes with the length 327 mm. The temperature was changed from 17.5 up to 25 ^{o}C by the even cooling and warming of tube and measured by the probe situated in the axis and half of the tube length. The influence of the thin wire probe was imponderable on the resonance quality of all tubes.

Fig 1 shows the dependencies of mode 1 and 5 frequencies for the tubes of diameter 41.8 mm. All mode 5 frequencies were normalized according to mode order. Mode 1 and 5 frequencies with tube lengths 656 and 327 mm were normalized once more to length 985 mm. It enabled to display all dependencies in only one frequency resolution.

For ideal conditions all dependencies should be identical. The parallel shift between the

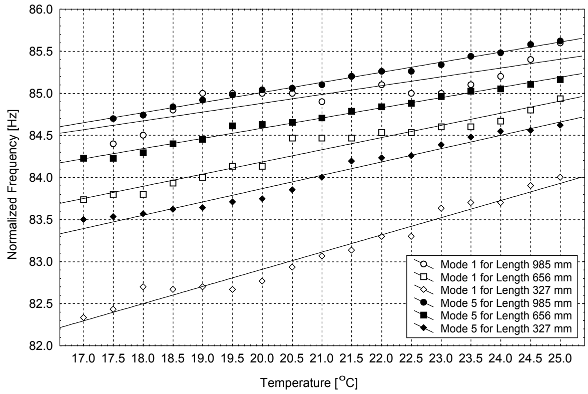

Fig. 1 Dependencies of resonance modes 1 and 5 frequencies on air column temperature in three tubes

dependencies with the same length reflects inharmonicities of resonance modes, shift between the dependencies in the same mode for different lengths shows different length correction, and different slope of all dependencies indicates different dependencies of mode frequency on temperature. The first effect is generally well-known, the second one will be discussed further as a topic of this paper and the third one reflects a complex of temperature dependencies in organ pipes known as out of tune organ due to great temperature changes.

The main measurement concentrated on monitoring of three dependencies: The dependence of length correction on the diameter of air column, the dependence of length correction on the length of air column and the dependencies of inharmonicities on the diameter and length of air column. The frequencies were measured for the tube length 985 mm up to resonance mode 19 (by non-existent even modes), for the length 656 mm up to mode 15 and for the length 327 mm up to mode 9. The acoustical length was calculated for all diameters of tubes in constant temperature. These dependencies are displayed in Fig. 2 for all three lengths in comparison with the classical relationships for the open-end correction 0.3D and 0.41D, which are valid for mode 1 frequency only. Fig. 2 demonstrates, that in mode 1, which determines the pitch of the pipe tone, the

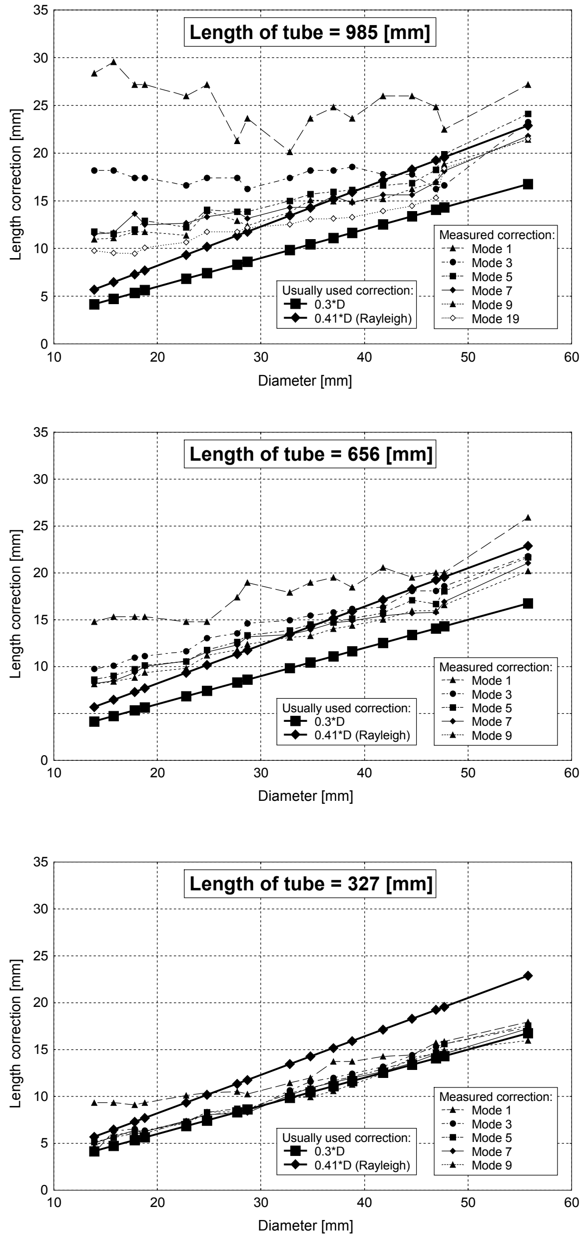

Fig. 2 Dependencies of open end length correction on air column diameter for some resonance modes

significant extension of correction over the above-listed formulas occurs up especially with small diameters. With the increasing frequency of the mode the dependence becomes rather linear and simultaneously edges to the classical formulas.

The dependencies of length correction on the tube length are shown in Fig. 3. There are the values for resonance mode 3 and 5, which were normalized to values of mode 1. The oscillated curve of mode 1 is interleaved

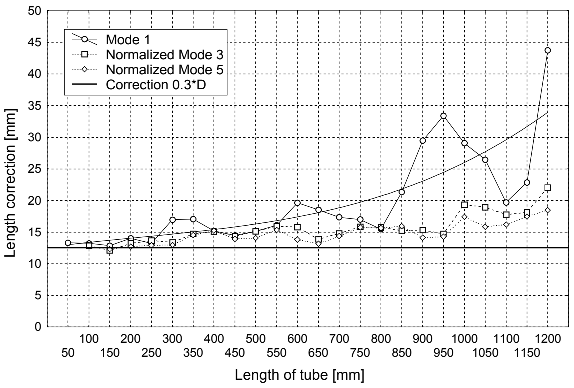

Fig. 3 Dependencies of open-end length correction on the tube length with diameter 41.8 mm for three resonance modes

with the polynomial of second order. The horizontal line represents the usual correction 0.3D independent of the tube length.

The relation between the relative inharmonicity of frequency position of the higher resonance modes and the tube length and diameter is displayed in Fig. 4. The inharmonicity is calculated as the relative deviation (%) from integer multiple of the mode 1 frequency.

Discussion

The results of incurred measurements indicate at more complex relationships between the open-end length correction and the diameter and mechanical length of cylindrical tube. A quite independent problem is the actual temperature influence of air column resonance properties, not solved in this paper. The well-known formulas for the open-end length correction considerably simplify the whole acoustical length question and represent greater material reserve in the organ pipe length design than a mere physical solution. Can be expected that in long and narrow cylindrical tubes make capillary effects cause opposite dependencies, for example the correction extending with the use of small diameters and long lengths of tube. There is no length correction independence of the mechanical length of tube, no known simple explanation of correction value oscillation in Fig. 3 either. Also evident is the very complicated dependence of all resonance mode inharmonicities on air column diameter than is generally supposed.

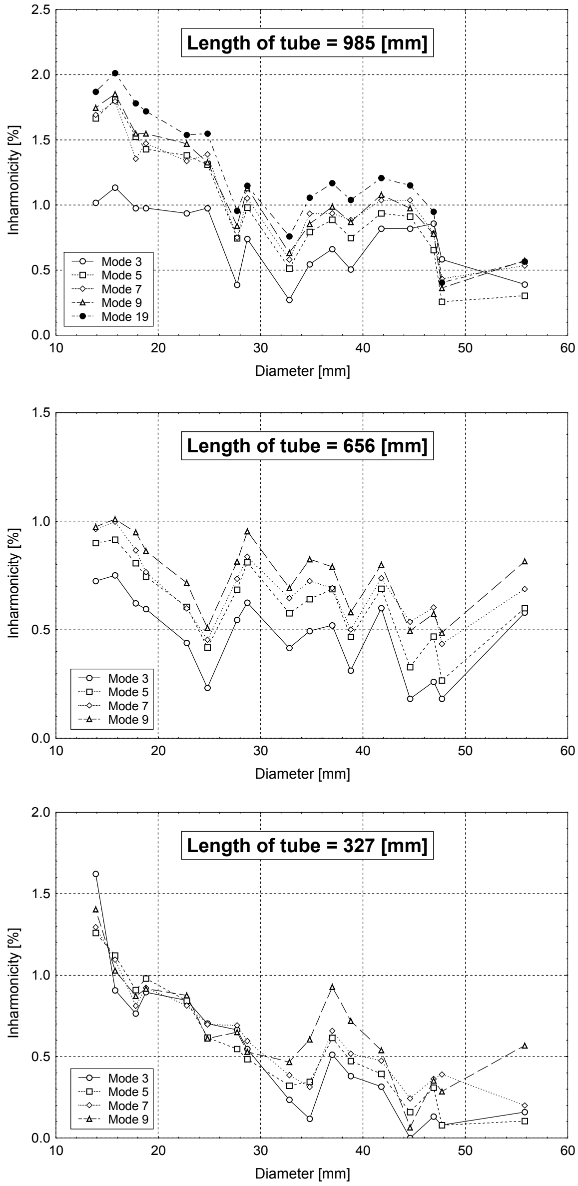

Fig. 4 Dependencies of inharmonicity on air column diameter for some resonance modes

When instead of the tube diameter only the diameter to length ratio is plotted, see Fig. 2, afterwards two tendencies (shown as lines in Fig. 5) are apparent in all dependencies. A notional explanation of two (at minimum) different mechanisms of air column oscillation is offered here: for the length 985

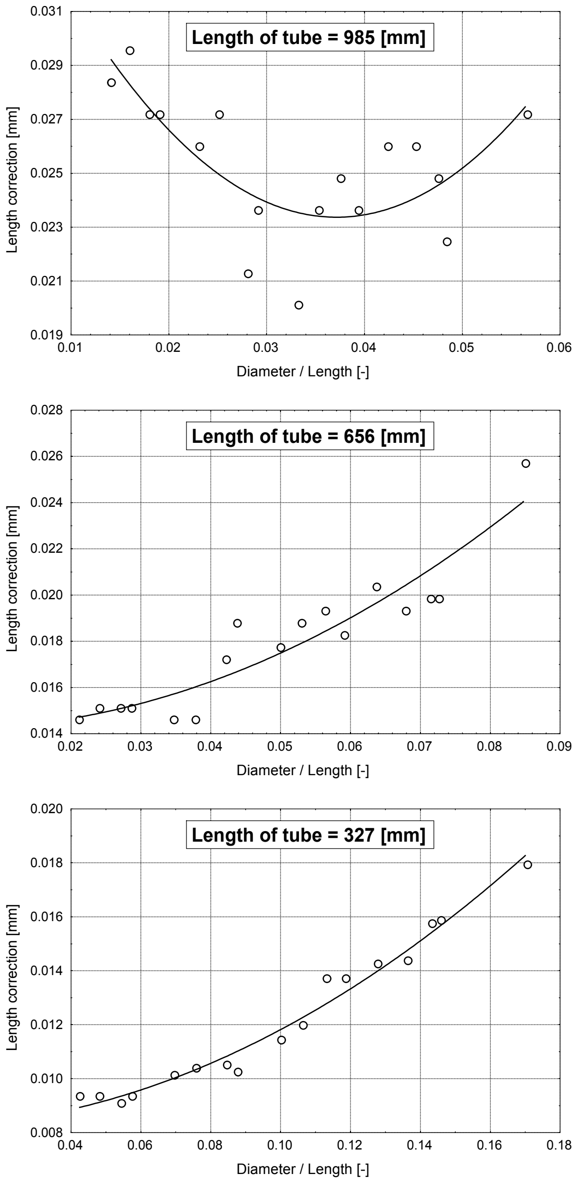

Fig. 5 Dependencies of open-end length correction on the ratio of diameter to length for resonance mode 1

mm: capillary for low ratios and non-capillary for high ratios of diameter to tube length.

Conclusions

As in any "acoustical" case as well, here rules applied in practice are subject to with many errors, nevertheless still survive. The acquired results open up many new theoretical and practical questions not only about the real acoustical length and its dependencies but also about acoustical measurement methods and the reliability and validity of the results.

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